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## **Role of algebra and number theory in the professional competence formation of future mathematics teachers**

This article describes the functions of algebra and number theory in the professional training of bachelors of pedagogical education in the specialty 5B010900 — «Mathematics». Algebra and number theory play an important role in the preparation of the future mathematics teacher, and above all, in its subject matters. Currently, particularly high demands are placed on the preparation of the future mathematics teacher. The success of student learning depends on how high-quality and comprehensive it will be. The main purpose of mastering this discipline is to improve the skills of working with numerical units; preparing for the deliberate use of the fundamentals of number theory, for example, such as the theory of divisibility, comparison theory, etc. The objectives of this subject can be attributed to the arming of students with theoretical knowledge, and the application of this knowledge in solving practical problems, and the formation of ideas about methods of mathematics as a universal language of science and technology. The discipline «Algebra and number theory» should also impart to students the skills of working with mathematical units, mathematical rigor of thinking, which is very necessary for work in the field of mathematics and other exact and natural sciences, and which they, as teachers, will develop among schoolchildren.

*Keywords:* competence, Professional competence, education, bachelor, number theory, vocational training.

All over the world, including in the Republic of Kazakhstan, global changes are taking place in the education system. These changes affected all levels of education, primarily the system of higher professional education. The Higher School of Kazakhstan, including pedagogical universities, has shifted to new standards.

These new standards meet the ideas of the competence approach, which defines the target orientation of the educational process to the formation of certain competencies that reflect a person's willingness to act in specific situations.

In the system of higher professional education, the initial task is the development of teacher education, and in particular the training of mathematics teachers.

The changes that have affected school mathematics education require different training from mathematics teachers. The school is waiting for a modern teacher who is able to solve the tasks set before the school. A modern mathematics teacher should possess innovative learning technologies that will equip students not only with subject knowledge, skills and abilities, but also personally significant qualities.

The training of a future mathematics teacher is a complex and multifaceted process, which is focused on the formation of professionally significant personal qualities, pedagogical abilities, competencies that meet the qualification requirements of the State educational standard of higher professional education.

*Professional competence* is understood as a combination of professional and personal qualities necessary for successful pedagogical activity. Professional competence includes key, basic and special competencies. The features of the formation of professional competence in the process of teaching mathematics teachers at a university are determined by the specifics of teacher training, the peculiarity of mathematical knowledge and future professional activities. Considering the professional competence formed by the future mathematics teacher within the framework of the university education system, R.M. Aslanov, A.V. Sinchukov, V.A. Testov distinguish three components of competency:

Therefore, these changes make it necessary to improve the skills and professionalism of the teacher, that is, his professional competence. Before we talk about the professional competence of the teacher, let us turn to the basic concepts of «competencies» and «competence»:

- informative (the availability of special mathematical knowledge);
- technological (knowledge of methods, techniques of teaching mathematics);
- personal (possession of personality traits necessary for a specialist in this profession).

Therefore, today any specialist needs to have a certain set of competencies [1].

The success of pedagogical activity largely depends on the skills and ability of each teacher to mobilize their own efforts for systematic mental work, rationally build their activities, manage their emotional and psychological state to use their potential, to show creative activity.

The problem of forming a future mathematics teacher is solved not only in the process of studying general pedagogical disciplines, but also should be systematically implemented in the process of studying mathematical disciplines. One of such fundamental disciplines for a mathematics teacher is algebra and number theory. In training a mathematics teacher, not only the understanding of the logical sources of science, its history, but also its connection with practice and application is brought to the forefront. One of the first motivations for studying a subject is knowledge of the area of its use. The origin, emergence and application of algebraic structures can be found in every mathematical discipline, that is, in the subject area of a future mathematics teacher. In addition, when it comes to training future teachers, then higher mathematics should not only give a general mathematical education, but also answer completely specific questions of the school course. Therefore, among the main ideas of any course in higher mathematics, the idea of its connection with elementary mathematics should find its place. And here it is already difficult to overestimate the role of such algebraic structures as groups, rings and fields, which in fact owe their appearance to the development of the concept of number and arithmetic operations and which are extremely necessary in the study of number theory and number systems that are directly related to school mathematics.

Algebra and number theory is one of the main components in the formation of professional competencies of pedagogical education bachelors in the specialty 5B010900 — «Mathematics». Its special role is determined by the diversity and significance of its main functions [2].

*Methodological function.* Methodological function in science is determined not only by cognitive, but also by practical activity. In the first mathematical courses, future teachers of mathematics first become acquainted with such a powerful method of mathematical knowledge of the world as mathematical modeling. Mathematics, like other sciences, studies the real world. However, it has the property of universal applicability. The modern stage of development of many sciences is characterized by the active use of mathematical models. The concepts of mathematical modeling are currently acquiring general cultural and general educational value and open up opportunities for students to form ideas about the role of models and modeling not only in mathematics, but also in physics, chemistry, biology, ecology, geography, economics, etc. In the course «Algebra and number theory», students study, in particular, elements of the theory of divisibility, the theory of comparisons, Diophantine equations, matrices, systems of linear equations illustrating various mathematical models.

The modern content of education, aimed at the formation of a scientific worldview, is characterized by trends towards encyclopaedism, integration of knowledge, humanistic and aesthetic orientation. Therefore, the following function — *worldview*.

The scientific worldview today is seen as humanistic. It can be defined as a generalized system of attitudes, beliefs, ideals, through which a person expresses his attitude towards the natural and social environment that surrounds him.

Algebra and number theory plays a significant role in the formation of a future teacher of a scientific worldview, a certain level of mathematical and methodological culture. This is especially true for understanding the essence of the applied orientation of teaching mathematics. The applied orientation of number theory consists, in particular, in that its concepts and methods are the theoretical foundations of cryptography and coding theory, algebra and number theory have numerous practical applications in economics, management, planning and optimization, number-theoretic problems are found in many areas human activity [3].

The application of the number-theoretical apparatus in solving problems of other academic disciplines, the establishment of interdisciplinary and inter-blocking relations contains another important ideological aspect: the existence of interdisciplinary and inter-blocking links is an objective regularity, reflecting the interaction of natural science and humanitarian cultures, the interrelation of real world phenomena, thereby contributing forming a holistic picture of the world.

*Next function is developmental.* The priority of the developmental function of education highlights the problem of the formation and development of students' thinking.

Number theory contributes to the overall development of the student, has a significant impact on the formation of various forms of thinking, in particular, logical and algorithmic. The solution of number-theoretic problems, varied both in form and content, the compilation of their own problems, their collective discussion and solution in practical classes allows for systematic work in this direction, starting from the first days of students training in the pedagogical university.

In addition, the technology of teaching the theory of numbers has an impact on the formation of the figurative and associative thinking of a student. Building interdisciplinary, inter-cycle and inter-block relations of special disciplines with humanities and pedagogical disciplines, saturating the content of mathematical disciplines with visual sign-symbolic and graphic, historical, literary, artistic and other images is one of the effective ways to implement the functions of development of the image-associative component of thinking of a teacher of mathematics.

*Humanizing function.* Many sections of the number theory allow the use of historical, literary, linguistic and other reminiscences.

Among the components of the humanitarian potential of the theory of numbers in a pedagogical university is its professional pedagogical orientation. This allows the realization in teaching of the principle of binarity — an adequate combination of mathematical and methodical lines.

For example, studying the elements of the theory of divisibility, properties of primes, Diophantine equations allows the future teacher to see the internal logical structure of school mathematics textbooks, to take a fresh look at the methods of proving theorems already known from school practice.

Interdisciplinary and especially intercycle links of the theory of numbers allow to refer to the cultural achievements of human civilization. Number theory is not only a powerful tool for solving applied problems, but also an element of general culture. Its practical applications are inexhaustible. Many sections of the number theory have significant humanitarian potential, as discussed above. Opportunities for implementing the cultural course function are: consideration of the development of basic concepts of number theory in the context of a cultures dialogue, the use of historical, literary, artistic images, sayings of famous scientists, etc. In particular, in the introductory lecture to the course of number theory it is appropriate to link its development with the history of culture. The stages of the historical development of the theory of numbers allow us to refer to the history and culture of ancient Greece and the Ancient East, Medieval Europe, the Renaissance and the New Age [4].

*Axiological function.* The educational training system should be aimed at the formation of a future teacher. Teaches not so much the content voiced by the teacher, but, above all, his personality. The system of knowledge that the teacher sets out, the educational effects that he exercises, are perceived in the refraction of his individuality. In pedagogical university pedagogical ethics should arise, value orientations of the future teacher should be formed. The formation of the axiological culture of a teacher is greatly facilitated by historical and cultural knowledge, especially personalistic. Turning to the images of mathematicians of the past, we not only increase interest in the subject, but also form the value system of the future teacher. In particular, consideration of the contribution of domestic scientists to number theory allows the formation of axiological orientations of future mathematics teachers to national values. So, for example, when studying number theory, students get acquainted with the names of L. Euler, V.Ya. Bunyakovsky, P.L. Chebysheva, I.M. Vinogradova, A.G. Postnikova, N.P. Romanov and many other domestic scientists.

*Creative function.* The need for research activities develops interest in patterns, teaches us to see the beauty and harmony of human thought. All these are the most important elements of the general culture. Any creative process begins with the formulation of a hypothesis. With appropriate training, mathematics, being a good school of building and testing hypotheses, teaches comparing different hypotheses, finding the best option, setting new tasks, looking for ways to solve them. In addition, it also develops the habit of methodical work, without which no creative process can be conceived. Maximizing the possibilities of human thinking, mathematics is its highest achievement. It helps a person in self-awareness and the formation of his character. All this fully applies to the course of the number theory. Learning the art of solving a problem provides an opportunity for students to develop a certain mindset. Sections of the theory of numbers contain a large number of problems, diverse in content and methods of solution. In studying course, students perform various individual tasks: to invent a task or a cycle of tasks, to develop a package of test tasks for the module of the discipline, etc.

*Successive propaedeutic function.* The number theory is one of the branches of mathematics most closely associated with the school course of mathematics, so when studying the number theory, the question of the implementation of successive links between school and university is particularly acute. This function allows you to implement the principle of the leading idea — the relationship of number theory with the school course. The content of the discipline «Number Theory» includes some questions of the school course. In particular, the numerical line of the school course of mathematics continues in the course of the theory of numbers by the study of prime and composite numbers and the theory of divisibility, the line of equations and

inequalities is presented in the theory of numbers by elements of the theory of comparisons and Diophantine equations, etc.

*The integration function* of number theory is the ability of students to form an integrated system of knowledge about the world around them. This is achieved through the integration of knowledge from various disciplines, providing an understanding of life phenomena, the place and role of man in cognition and transformation of the world. Integration is one of the directions for the implementation of student-centered education [5].

This function, in our opinion, is dominant in the system of fundamental functions of the theory of numbers, since all of the above functions are to a greater or lesser extent realized through its integration links. Designing the content of number theory on an integrative basis helps to overcome the disciplinary disunity of scientific knowledge, creates the conditions for the profile differentiation of learning.

As an example, let us consider the presentation of the section «Linear space». The mathematical object to be studied is «linear space». Its structure is created from various elements, for example, «matrices», «functions», «geometric vectors». The axioms of linear space are the backbone connections. If in this system we consider polynomials as elements, then we will talk about the form of the system (subsystem) as the linear space of polynomials. If the elements are matrices, then the linear space of matrices is studied. Specific mathematical methods allow you to find the basis, the dimension of the subsystem under study, work with transformations (linear operators) within its particular form. The study of this discipline on the basis of system analysis opens up important methodological concepts for students (system, structure, system-forming links, etc.). System analysis presents the structure of the system as an invariant concept. Mathematics through the description of various algebraic structures (the structure of linear space, fields, groups) demonstrates their specific expression depending on the elements and system-forming links [6].

Linear space and its specific types are widely used in analytical geometry, mathematical analysis, and differential equations. In the model under consideration, such interdisciplinary connections are opened by the student through the introduction of applied problems. Thus, a plot problem from a physics course is proposed, which boils down to a system of linear inhomogeneous equations. The resulting solution of the system is structured and disclosed through interrelations with the solution space of the corresponding homogeneous system. The study of the properties of the linear space of continuous functions lays the foundation for the successful assimilation of the theory of differentiation, integration, and the theory of series.

The content component of the professional competence of the future mathematics teacher poses one of the tasks in teaching mathematical disciplines is the connection of a particular course and the corresponding school subject. For example, consideration of a specific type of «linear space» system implies the ability to find the coordinates of vectors and their lengths on the basis of school planimetry. When finding the basis of space, methods of solving systems of equations known since school are used. The study of a quadratic form with a parameter specified in a linear space is based on the ability to work with inequalities [7].

Abstract methodological terms are filled with specific content. As for mathematical concepts, for students they are divided into «known» and «new». Notions on elementary algebra and geometry, for example, equation, inequality, function, etc., are among the first (known) ones. New for students are the general terms of higher mathematics and the concepts of a specific section of analytical geometry, mathematical analysis, probability theory, etc. In this case, rigorous mathematical definitions are given (determinant of the matrix, linear space, basis, linear operator, limit of a function, etc.). The properties of the mathematical system under study are revealed by means of theorems and special mathematical methods. In this context, higher mathematics appears in conjunction with the methodology of knowledge, shows the movement of mathematical science from elementary knowledge (school level) to higher mathematics (university level).

In the process of studying this section, the development of the technological component of the professional competence of a future mathematics teacher can be carried out through the content and methods of solving professionally directed algebraic problems in practical classes. In the studied training model, a special place is given to the system of educational and cognitive tasks, which includes both standard mathematical tasks and tasks with professional content, tasks for mastering general scientific and particular methods. The formation of abilities to use various methods equips a future university graduate with a powerful apparatus for successfully solving professional problems. The classification of types of tasks is based on the types of activities formed during their solution, which is directly related to the formation of the learner's competencies. Tasks for mastering methodological knowledge and general methods of cognition (system analysis, mathematical modeling, synthesis, etc.), mathematical knowledge and skills are distinguished [8]. A separate class is the tasks of the research plan aimed at developing the ability to independently study new material,

to seek a solution to the problem. A public discussion of such issues at seminars allows students to master the skills of working in a team, to argue and defend their point of view.

Let us give some examples of educational — cognitive tasks.

Task 1. «Given a set of matrices of a certain type. Select the elements of the set, establish a backbone connection. Will the given set of matrices has the structure of a linear space (subspace)?».

Task 2. «Given a set of vectors of a certain type. Check that the given set of vectors has the structure of a linear subspace in the space of arithmetic vectors. Distinguish the basis of the subspace».

Task 3. «Among several given sets of arithmetic vectors, select objects with the structure of a linear subspace. Find the basis and dimension of the subspace, add the selected basis to the basis of the whole space».

Thus, the fundamental functions of number theory are significant and diverse, which determines its role in the system of professional training of teacher education bachelors in the specialty «Mathematics». The role, place and content of algebraic courses in the training of future mathematics teachers require clarification and reevaluation against the background of ongoing changes in educational plans. This is important for the formation of a competent mathematician, and for the training of a qualified mathematics teacher.

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### **Болашақ математика мұғалімдерінің кәсіби құзыреттіліктерін қалыптастырудағы алгебра және сандар теориясының ролі**

Мақалада 5B010900 — «Математика» мамандығы бойынша бакалаврлардың кәсіби даярлығындағы алгебра және сандар теориясының функциялары қарастырылған. Алгебра және сандар теориясы пәні болашақ математика мұғалімін даярлауда, әсіресе оның пәндік құрамдас бөлігі ретінде маңызды роль атқарады. Қазіргі уақытта болашақ математика мұғалімін даярлауға ерекше жоғары талаптар қойылады. Оқушыларды оқытудың табыстылығы оның қаншалықты сапалы және жан-жақты болатынына байланысты. Берілген пәнді игерудің негізгі мақсаты сандық объектілермен жұмыс жасау дағдыларын жетілдіру болып табылады; сандық теория негіздерін саналы түрде қолдануға дайындау, мысалы, бөлінгіштік, салыстыру теориясы және т.б. Пәннің міндеттеріне студенттерді теориялық біліммен қаруландыру, алған білімдерін практикалық сипаттағы есептерді шешуге қолдану және математика әдістері туралы түсінікті ғылым мен техниканың әмбебап тілі ретінде қалыптастыруды жатқызуға болады. «Алгебра және сандар теориясы» пәні студенттерге математика мен басқа да нақты және жаратылыстану ғылымдары саласында жұмыс жасауда қажетті математикалық қатан ойлауды, математикалық объектілермен жұмыс істеу дағдыларын дарытады және олар оны болашақ педагог ретінде оқушыларда қалыптастыратын болады.

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## Роль алгебры и теории чисел в формировании профессиональных компетенций у будущих учителей математики

В статье рассмотрены функции алгебры и теории чисел в профессиональной подготовке бакалавров педагогического образования по специальности 5В010900 — «Математика». Алгебра и теория чисел играют важную роль в подготовке будущего учителя математики и, прежде всего, в ее предметной составляющей. В настоящее время к подготовке будущего учителя математики предъявляются особенно высокие требования. От того, насколько качественной и всесторонней она будет, зависит успешность обучения учеников. Основная цель освоения данной дисциплины заключается в совершенствовании навыков работы с числовыми объектами; подготовка к осознанному использованию основ теории чисел, например, таких как теории делимости, сравнения и т.д. К задачам данной дисциплины можно отнести и вооружение студентов теоретическими знаниями, применение этих знаний при решении задач практического характера и формирование представления о методах математики как универсального языка науки и техники. Дисциплина «Алгебра и теория чисел» также должна привить студентам навыки работы с математическими объектами, математическую строгость мышления, весьма необходимую для работы в области математики и других точных и естественных наук и которую они, как педагоги, будут вырабатывать у школьников.

*Ключевые слова:* компетенция, профессиональная компетенция, образование, бакалавр, теория чисел, профессиональная подготовка.

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